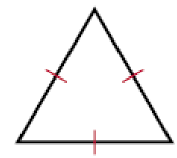
Attendance Problems.

1. Find each angle measure.



True or False. Explain your choice.

2. _____ Every equilateral triangle is isosceles.

3. _____ Every isosceles triangle is equilateral.

- I can prove theorems about isosceles and equilateral triangles.
- I can apply properties of isosceles and equilateral triangles.

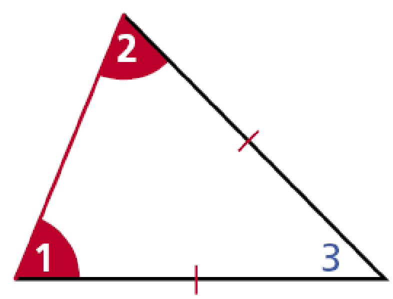
Vocabulary	
vertex angle	base
legs of an isosceles triangle	base angles

Common Core: CC.9-12.G.CO.10 Prove theorems about triangles.

Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the **legs**. The **vertex angle** is the angle formed by the legs. The side opposite the vertex angle is called the **base**, and the **base angles** are the two angles that have the base as a side.

<3 is the vertex angle.

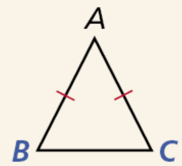
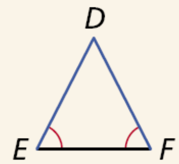
<1 and <2 are the base angles.



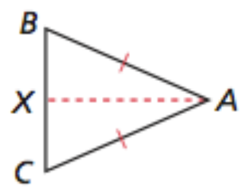
Q: What did the frowning student say to his teacher?
A: “I’ve got more problems than a math book.”

"The urge to destroy is also a creative urge." -- *Bakunin*

Theorems — Isosceles Triangle

THEOREM	HYPOTHESIS	CONCLUSION
4-8-1 **Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite the sides are congruent.		$\angle B \cong \angle C$
4-8-2 **Converse of Isosceles Triangle Theorem** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.		$\overline{DE} \cong \overline{DF}$

Isosceles Triangle Theorem



Given: $\overline{AB} \cong \overline{AC}$
Prove: $\angle B \cong \angle C$

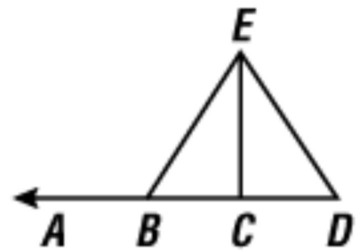
Proof:

Statements	Reasons
1. Draw X, the mdpt. of $\overline{BC}$.	**1.** Every seg. has a unique mdpt.
2. Draw the auxiliary line $\overline{AX}$.	**2.** Through two pts. there is exactly one line.
3. $\overline{BX} \cong \overline{CX}$	**3.** Def. of mdpt.
4. $\overline{AB} \cong \overline{AC}$	**4.** Given
5. $\overline{AX} \cong \overline{AX}$	**5.** Reflex. Prop. of $\cong$
6. $\triangle ABX \cong \triangle ACX$	**6.** SSS *Steps 3, 4, 5*
7. $\angle B \cong \angle C$	**7.** CPCTC

Reading Math

The Isosceles Triangle Theorem is sometimes stated as “Base angles of an isosceles triangle are congruent.”

Video Example 1.



C is the midpoint of $\overline{BD}$.

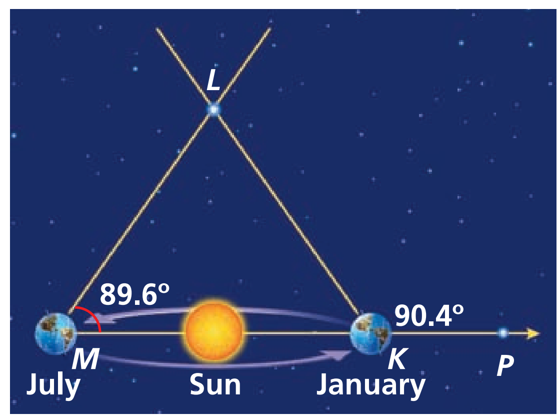
Given: $m\measuredangle ABE = 105°$

$m\measuredangle CDE = 75°$

Explain why BE = ED.

1 *Astronomy Application*

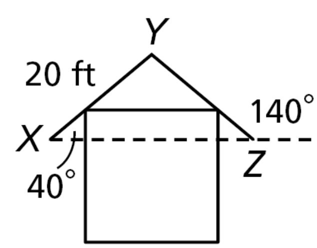
The distance from Earth to nearby stars can be measured using the parallax method, which requires observing the positions of a star 6 months apart. If the distance *LM* to a star in July is 4.0×10^{13} km, explain why the distance *LK* to the star in January is the same. (Assume the distance from Earth to the Sun does not change.)



Not drawn to scale

$m\angle LKM = 180 - 90.4$, so $m\angle LKM = 89.6°$. Since $\angle LKM \cong \angle M$, $\triangle LMK$ is isosceles by the Converse of the Isosceles Triangle Theorem. Thus $LK = LM = 4.0 \times 10^{13}$ km.

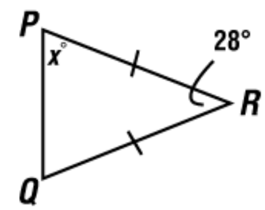
Example 1. The length of *YX* is 20 feet. Explain why the length of *YZ* is the same.



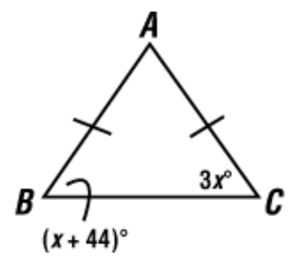
4. <u>Guided Practice</u>. If the distance from Earth to a star in September is 4.2×10^{13} km, what is the distance from Earth to the star in March? Explain your answer.

<u>Video Example 2</u>.

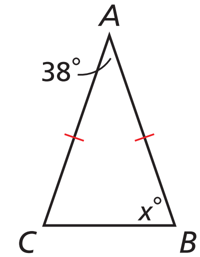
A. Find $m\measuredangle Q$.



B. Find $m\measuredangle C$.



2 Finding the Measure of an Angle



Find each angle measure.

A $m\angle C$

$m\angle C = m\angle B = x°$ — *Isosc. △ Thm.*

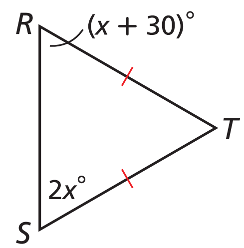
$m\angle C + m\angle B + m\angle A = 180$ — *△ Sum Thm.*

$x + x + 38 = 180$ — *Substitute the given values.*

$2x = 142$ — *Simplify and subtract 38 from both sides.*

$x = 71$ — *Divide both sides by 2.*

Thus $m\angle C = 71°$.



B $m\angle S$

$m\angle S = m\angle R$ — *Isosc. △ Thm.*

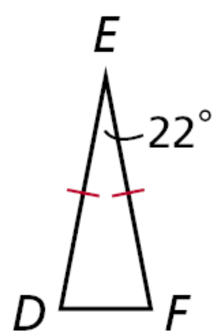
$2x° = (x + 30)°$ — *Substitute the given values.*

$x = 30$ — *Subtract x from both sides.*

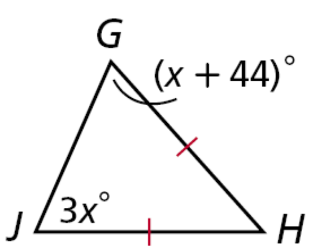
Thus $m\angle S = 2x° = 2(30) = 60°$.

Example 2.

A. Find $m\measuredangle F$.

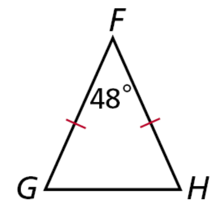


B. Find $m\measuredangle G$.

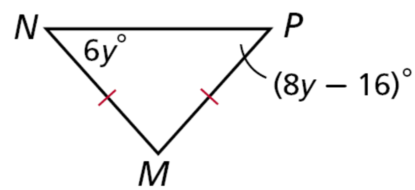


<u>Guided Practice</u>.

5. Find m<H.

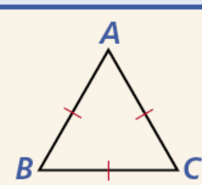


6. Find m<*N*.

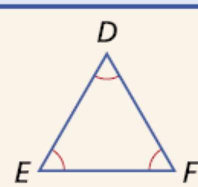


The following corollary and its converse show the connection between equilateral triangles and equiangular triangles.

Corollary 4-8-3 **Equilateral Triangle**

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equilateral, then it is equiangular. (equilateral △ → equiangular △)		$\angle A \cong \angle B \cong \angle C$

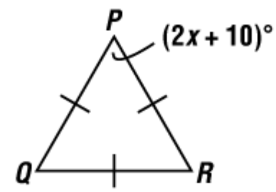
Corollary 4-8-4 **Equiangular Triangle**

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equiangular, then it is equilateral. (equiangular △ → equilateral △)		$\overline{DE} \cong \overline{DF} \cong \overline{EF}$

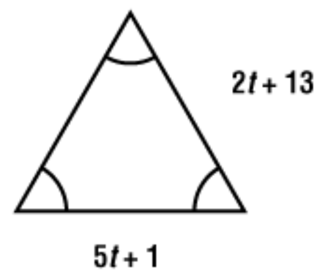
Pre-AP Geometry 4-9 Study Guide: Isosceles and Equilateral triangles (pp 285-291)

Video Example 3.

A. Find the value of x.



B. Find the value of t.

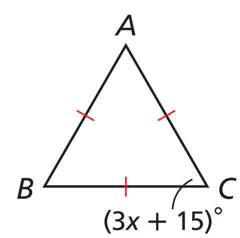


3 Using Properties of Equilateral Triangles

Find each value.

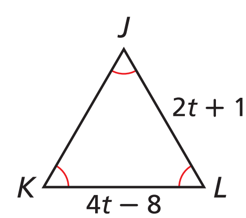
A x

$\triangle ABC$ is equiangular.	*Equilateral △ → equiangular △*
$(3x + 15)^\circ = 60^\circ$	*The measure of each ∠ of an equiangular △ is 60°.*
$3x = 45$	*Subtract 15 from both sides.*
$x = 15$	*Divide both sides by 3.*



B t

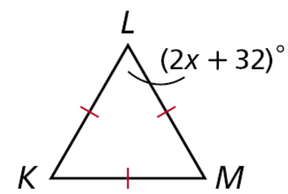
$\triangle JKL$ is equilateral.	*Equiangular △ → equilateral △*
$4t - 8 = 2t + 1$	*Def. of equilateral △*
$2t = 9$	*Subtract 2t and add 8 to both sides.*
$t = 4.5$	*Divide both sides by 2.*



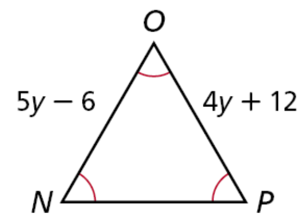
Pre-AP Geometry 4-9 Study Guide: Isosceles and Equilateral triangles (pp 285-291)

<u>Example 3</u>.

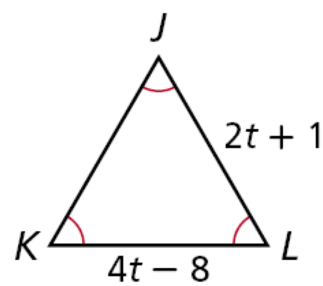
A. Find the value of x.



B. Find the value of y.



7. <u>Guided Practice</u>. Find the value of JL.



<u>4-9 Isosceles & equilateral triangles</u> *(p 289)* 12, 13-19 odd.

Remember!

A coordinate proof may be easier if you place one side of the triangle along the x-axis and locate a vertex at the origin or on the y-axis.

<u>Video Example 4</u>.

$\triangle ABC$ is right isosceles.

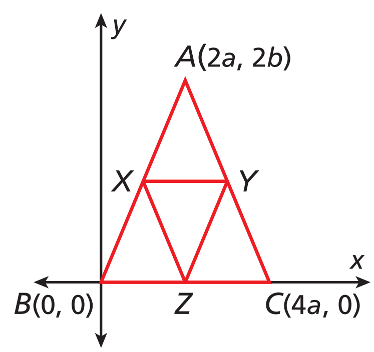
Given: D is the midpoint of $\overline{AC}$.

$\overline{AB} \cong \overline{BC}$

Prove: $\triangle BDC$ is isosceles.

4 Using Coordinate Proof

Prove that the triangle whose vertices are the midpoints of the sides of an isosceles triangle is also isosceles.



Given: $\triangle ABC$ is isosceles. X is the mdpt. of $\overline{AB}$. Y is the mdpt. of $\overline{AC}$. Z is the mdpt. of $\overline{BC}$.

Prove: $\triangle XYZ$ is isosceles.

Proof:

Draw a diagram and place the coordinates of $\triangle ABC$ and $\triangle XYZ$ as shown. By the Midpoint Formula, the coordinates of X are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) = (a, b)$, the coordinates of Y are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) = (3a, b)$, and the coordinates of Z are $\left(\frac{4a+0}{2}, \frac{0+0}{2}\right) = (2a, 0)$.

By the Distance Formula, $XZ = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$, and $YZ = \sqrt{(2a-3a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$.

Since $XZ = YZ$, $\overline{XZ} \cong \overline{YZ}$ by definition. So $\triangle XYZ$ is isosceles.

<u>Example 4</u>.

Isosceles $\triangle ABC$

Given: X is the midpoint of $\overline{AB}$

Y is the midpoint $\overline{AC}$

Prove: $XY = \frac{1}{2}BC$

8. <u>Guided Practice</u>. The coordinates of isosceles ΔABC are $A(0, 2b)$, $B(-2a, 0)$, and $C(2a, 0)$. X is the midpoint of AB, and Y is the midpoint of AC. Prove ΔXYZ is isosceles.

<u>4-9 Isosceles & equilateral triangles</u>

- *(p 289)* 12, 13-21 odd, 28, 30.
- 5B Ready to Go On pretest & posttests.